An Exact Value for Avogadro’s Number Redux

My friend Ted Hill has recently communicated a paper by him, Jack Miller and me to the Physics Archives. It is an extension and reemphasis of our earlier American Scientist paper and is especially devoted to a new definition for the kilogram standard. In the newer paper the reader is asked to imagine a cubic array of carbon atoms. As we noted on page 4 of the American Scientist paper, carbon does not have an extended cubic structure. Both carbon (as diamond) and silicon make use of $sp^3$ orbitals to form tetrahedral bonds between neighboring atoms.

As explained in American Scientist instead of finding the $n$ in

$$n^3 = N_A$$

One must find the $k$ in

$$8k^3 - 18k^2 + 15k - 4 = N_A$$

in which $N_A$ is Avogadro’s number. In Physics Archives we solved for $n$ and obtained

$$n = 84446889$$

This value gives $84446889^3 = 602214162464240016093369$ for $N_A$. The third triplet 162 is to be compared with 179. The triplet 179 is from the most recent NIST value for Avogadro’s number, $6.02214179 \pm 0.00000030 \times 10^{23}$.

In the $n$ formula $n$ is the number of atoms on the edge of an imagined perfect cubic array. In the $k$ formula there is a tetrahedrally arranged array that includes a cubic array of atoms having $k$ atoms on an edge. Other interstitial atoms complete the tetrahedral bonds and give rise to the additional terms in the $k$ formula. Crudely, the leading term says $k = \frac{n}{2}$ but $k$ will need to be somewhat larger than $\frac{n}{2}$ in order for the resulting value for Avogadro’s number to be near the current experimental value. Thus we find that $k = 42223446$ works well:
\[8 \times 42223446^3 - 18 \times 42223446^2 + 15 \times 42223446 - 4 = 602\,214\,194\,554\,987\,427\,447\,486\]

The third triplet 194 does better than did 162 above by just a touch.

In diamond the carbon atoms on a cube edge of the unit cell are 3.567 angstroms apart. This is not a bond length but a spacing in the extended tetrahedral array. Since there are 42223446 atoms on an edge, a “cubic” lump of diamond will have an edge of length

\[42223445 \times 3.567 = 1.50611028315 \text{ centimeters}.\]

This amounts to 12 grams of diamond (60 carats!). A kilogram is \(83\frac{1}{3}\) times more massive and greater in volume. The number of atoms on an edge and the length of an edge for a kilogram of diamond would increase by \(\sqrt[3]{83\frac{1}{3}} = 4.367902323\ldots\)

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