An Exact Value for Avogadro's Number Redux

My friend Ted Hill has recently communicated a paper by him, Jack Miller and me to the Physics Archives. It is an extension and reemphasis of our earlier *American Scientist* paper and is especially devoted to a new definition for the kilogram standard. In the newer paper the reader is asked to imagine a cubic array of carbon atoms. As we noted on page 4 of the *American Scientist* paper, carbon does not have an extended cubic structure. Both carbon (as diamond) and silicon make use of sp^3 orbitals to form tetrahedral bonds between neighboring atoms.

As explained in *American Scientist* instead of finding the n in

$$n^3 = N_A$$

One must find the k in

$$8k^3 - 18k^2 + 15k - 4 = N_A$$

in which N_A is Avogadro's number. In Physics Archives we solved for n and obtained

$$n = 84446889$$

This value gives $84446889^3 = 602\ 214\ 162\ 464\ 240\ 016\ 093\ 369$ for N_A. The third triplet 162 is to be compared with 179. The triplet 179 is from the most recent NIST value for Avogadro's number, $6.02\ 214\ 179 \pm 0.000\ 000\ 30 \times 10^{23}$.

In the n formula n is the number of atoms on the edge of an imagined perfect cubic array. In the k formula there is a tetrahedrally arranged array that includes a cubic array of atoms having k atoms on an edge. Other intersticial atoms complete the tetrahedral bonds and give rise to the additional terms in the k formula. Crudely, the leading term says $k = \frac{n}{2}$ but k will need to be somewhat larger than $\frac{n}{2}$ in order for the resulting value for Avogardro's number to be near the current experimental value. Thus we find that $k = 42\ 223\ 446$ works well:

$$8 \times 42223446^3 - 18 \times 42223446^2 + 15 \times 42223446 - 4$$
$$= 602\ 214\ 194\ 554\ 987\ 427\ 447\ 486$$

The third triplet 194 does better than did 162 above by just a touch.

In diamond the carbon atoms on a cube edge of the unit cell are 3.567 angstroms apart. This is not a bond length but a spacing in the extended tetrahedral array. Since there are 42223446 atoms on an edge, a “cubic” lump of diamond will have an edge of length

$$42223445 \times 3.567 = 1.50611028315 \text{ centimeters.}$$

This amounts to 12 grams of diamond (60 carats!). A kilogram is $83\frac{1}{3}$ times more massive and greater in volumn. The number of atoms on an edge and the length of an edge for a kilogram of diamond would increase by $\sqrt[3]{83\frac{1}{3}} = 4.367902323$ ...

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