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Hydrodynamic correlation functions for a nematic liquid crystal in a stationary state

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Abstract

We show that the general procedure developed by Fox and Uhlenbeck (Phys. Fluids 3 (1970) 1893) may be employed to describe the hydrodynamic fluctuations of a thermotropic nematic liquid crystal in equilibrium and steady states. We calculate explicitly the matrix of correlation functions for the transverse variables of a nematic film confined between two horizontal plates and subjected to a constant pressure gradient. We find that the light scattering spectrum and the intensity of the Rayleigh line are calculated from the orientation correlation function in equilibrium to first order in the pressure gradient. The shape and intensity of these lines deviate from their equilibrium values by amounts proportional to the imposed gradient, leading to an asymmetry in their height and intensity. It is shown that these effects may as large as 94.3% for a value of $\mid \nabla p \mid = 2.64 \times 10^{-2}$ atm/cm of the pressure gradient, suggesting that this effect might be observable.

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1. Introduction

The Landau and Lifshitz theory of hydrodynamic fluctuations [1] close to equilibrium has been put on a firm basis within the framework of the general theory of stationary Gaussian Markov processes by Fox and Uhlenbeck long ago [2,3]. This approach has matched the theory of Onsager and Machlup [4,5] with the approach of Landau and Lifshitz, for systems where the basic state variables, $\{a_i(\vec{r}, t)\}$, do not posses a definite time reversal symmetry. Although Fox and Uhlenbeck's scheme has been applied to simple fluids and their binary mixtures [6,7], its applications to other complex fluids are rather scarce.

In spite of the fact that the theory of fluctuations in nonequilibrium fluids was initiated in the late 70's [8–13] and pursued by many authors [14–19], still nowadays several questions concerning the nature of hydrodynamic fluctuations in stationary nonequilibrium states are of current active interest. One of these issues is the long-range character of these fluctuations, specially far away from instability points [20]. Thermal fluctuations in an equilibrium fluid always give rise to short-range equal time correlation functions, except close to a critical point. But when external gradients are applied, equal-time correlation functions can develop long-range contributions, whose nature is very different from those in equilibrium. For many models and systems in nonequilibrium states it has been shown theoretically that the existence of the so called generic scale invariance [21,22], is the origin of the long range nature of the correlation functions [23]. In the case of a simple fluid in a thermal gradient the structure factor, which determines the intensity of the Rayleigh scattering, diverges as q^{-4} for small values of the wave number q. This amounts to an algebraic decay of the density–density correlation function, a feature that has been verified experimentally [24–27].

However, in spite of the considerable interest in fluctuations about dissipative steady states of simple fluids during the last two decades, there are few similar studies for equilibrium or nonequilibrium stationary states of complex fluids. Among these the enhancement of concentration fluctuations in polymer solutions under external hydrodynamic and electric fields [28], or the case of a polymer solution subjected to a stationary temperature gradient in the absence of any flow [29], have been discussed. Also, the behavior of fluctuations about some stationary nonequilibrium states have been analyzed in the case of thermotropic nematic liquid crystals. Specific examples are the nonequilibrium situations generated by a static temperature gradient [30], a stationary shear flow [31] or by an externally imposed constant pressure gradient [32–34]. Although, these studies have considered only the particular case of transverse modes of the nematic, in the first two cases it was found that the nonequilibrium contributions to the corresponding light scattering spectrum were small, but in the case of a Poiseuille flow induced by an external pressure gradient the effect may be quite large. To our knowledge, however, at present there is no experimental confirmation of these effects, in spite of the fact that for nematics the scattered intensity is several orders of magnitude larger than for ordinary simple fluids.

The main purpose of this work is first, to show that the general procedure developed by Fox and Uhlenbeck may be employed to treat complex fluids systems like a thermotropic nematic liquid crystal in equilibrium and steady states. Secondly, for this system we calculate analytically all the correlation functions of the transverse state variables when the nematic is out of equilibrium and we evaluate their effect on the dynamic structure factor of the fluid. For this purpose the plan of the paper is as follows. In Section 2, we reconsider the model of a nematic thin film subjected to an externally imposed constant pressure gradient [32,33] and we show explicitly that the fluctuating linearized hydrodynamic equations may be recasted in the general form of Fox and Uhlenbeck. The entropy production for the nematic, the fluctuation–dissipation relations for the corresponding stochastic currents are also determined and the corresponding Fokker–Planck equation for the two point distribution function is written explicitly. The transverse hydrodynamic modes in equilibrium and for the steady state are calculated and the velocity, orientation and velocity–orientation correlation functions are derived as outlined in Section 3. From the orientation autocorrelation function we calculate the dynamic structure factor in the steady state and describe its alterations with respect to the equilibrium case. Finally, we close the paper by discussing some of the limitations, advantages and perspectives of this approach.

2. Model and basic equations

2.1. Linearized nematodynamic equations

The hydrodynamic description of uniaxial, thermotropic nematic liquid crystals is a well established subject and has been verified experimentally in detail [35–41]. Its generalization to include electrohydrodynamic effects has also been accomplished and has been triggered by the many electro-optic effects existing in these liquids which have produced a variety of applications in display devices [42,32], or for transport processes in these fluids [7,45].

The hydrodynamic state of the nematic is specified by the following variables: the velocity field, $\vec{v}(\vec{r},t)$, the unit vector defining the local symmetry axis (director), $\vec{n}(\vec{r},t)$; the mass density, $\rho(\vec{r},t)$, and the temperature, $T(\vec{r},t)$. If, as depicted in Fig. 1, the initial director's orientation $\hat{n}_0$ is chosen along the z axis, it is a preferred direction and the hydrodynamic variables may be divided into two independent sets, namely, transverse and longitudinal to $\hat{n}_0$ and the wave vector $\vec{k}$ [38]. The former set is $\{v_x(\vec{r},t),\ n_x(\vec{r},t)\}$, while the latter is $\{\rho(\vec{r},t),\ T(\vec{r},t),\ v_y(\vec{r},t),\ v_z(\vec{r},t),\ n_y(\vec{r},t)\}$. More precisely, Fig. 1 shows a planar homeotropic nematic cell of thickness d, where the molecules are oriented normal to a lower and upper plates. A constant external pressure gradient ∇p, acts along the plates and produces a stationary Poiseuille flow in the positive y direction

$$\vec{v}_0 = v_{0y}(z)\hat{y} \tag{1}$$

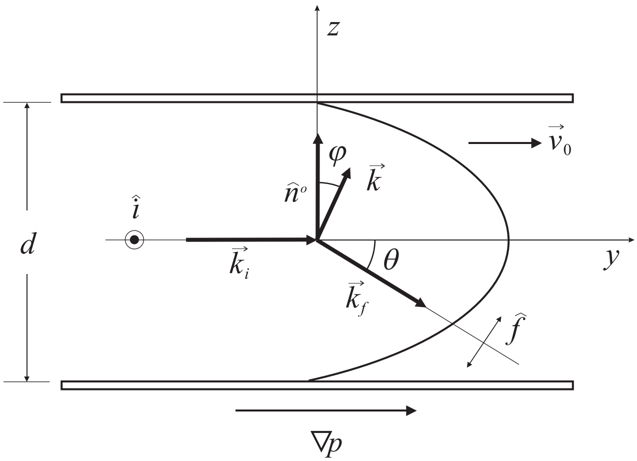

Fig. 1. Schematic representation of a plane homeotropic nematic cell. A constant pressure gradient is applied in the direction $\hat{y}$. θ is the scattering angle. $\vec{k} \equiv \vec{k}_i - \vec{k}_f$ is the scattering vector. The plane $(\vec{k}_i, \vec{k}_f)$ is the scattering plane and the angle between $\vec{k}_i$ and $\vec{k}_f$ is the (internal) scattering angle θ. $\hat{i}$ and $\hat{f}$ are the initial and final directions of the polarizations of the incident and scattered light. φ is the angle between $\vec{k}$ and the z axis.

with

$$v_{0y}(z) \equiv \frac{|\nabla p|}{2\nu_3}\left[z^2 - \left(\frac{d}{2}\right)^2\right] . \tag{2}$$

Here ν_3 is one of the five independent friction coefficients of a nematic in the notation of Harvard [35] and $\hat{y}$ denotes the unit vector along the y direction. It will be convenient to define a dimensionless pressure gradient

$$\gamma \equiv \frac{\rho_0 d^3}{\nu_3^2} \,|\nabla p|\,, \tag{3}$$

so that the flow field (1) is

$$v_{0y}(z) = \gamma(b_1 - b_2 z^2) \tag{4}$$

with $b_1 \equiv \nu_3/(8\rho_0 d)$ and $b_2 \equiv \nu_3/(2\rho_0 d^3)$. ρ_0 is the mass density in the reference equilibrium state.

For our purposes we shall only need the linearized hydrodynamic equations describing the dynamics of the small perturbations $\delta\rho(\vec{r},t) \equiv \rho(\vec{r},t) - \rho_0$, $\delta T(\vec{r},t) \equiv T(\vec{r},t) - T_0$, $\delta v_i(\vec{r},t) \equiv v_i(\vec{r},t)$, $\delta n_i(\vec{r},t) \equiv n_i(\vec{r},t) - n_{0i}$, resulting from spontaneous thermal fluctuations around the equilibrium state. In the Appendix the complete linearized set of hydrodynamic for these fluctuations is reviewed. We shall now rewrite Eqs. (123)–(126) by introducing the slightly modified set of state variables

$\{a_i(\vec{r},t)\}$ all possessing the same dimensionality

$$a_1(\vec{r},t) \equiv \frac{1}{(\rho_0)^{1/2}} \delta\rho(\vec{r},t) , \tag{5}$$

$$a_\alpha(\vec{r},t) \equiv \left(\frac{\rho_0}{A}\right)^{1/2} \delta v_\alpha(\vec{r},t), \quad \alpha = 2,3,4 , \tag{6}$$

$$a_5(\vec{r},t) \equiv \left(\frac{\rho_0 C}{T_0 A}\right)^{1/2} \delta T(\vec{r},t) , \tag{7}$$

$$a_\mu(\vec{r},t) \equiv \rho_0^{1/2} \delta n_\mu(\vec{r},t), \quad \mu = 6,7 , \tag{8}$$

where the equilibrium quantities A and C have been defined in the Appendix. Then Eqs. (123)–(126) become

$$\frac{\partial}{\partial t} a_1 + A^{1/2} \frac{\partial}{\partial x_\alpha} a_\alpha = 0 , \tag{9}$$

$$\begin{aligned} &\frac{\partial}{\partial t} a_\alpha + A^{1/2} \frac{\partial}{\partial x_\alpha} a_1 - \frac{1}{\rho_0} v^0_{\alpha j \beta l} \frac{\partial^2}{\partial x_j \partial x_l} a_\beta + \frac{B}{\rho_0} \left(\frac{T_0}{C}\right)^{1/2} \frac{\partial}{\partial x_\alpha} a_5 \\ &\quad + \frac{1}{2\rho_0 A^{1/2}} \lambda^0_{kj\alpha} \kappa^0_{kj\mu l} \frac{\partial^3}{\partial^2 x_j \partial x_l} a_\mu = 0 , \end{aligned} \tag{10}$$

$$\frac{\partial}{\partial t} a_5 + \frac{B}{\rho_0} \left(\frac{T_0}{C}\right)^{1/2} \frac{\partial}{\partial x_\alpha} a_\alpha - \frac{1}{\rho_0 C} \kappa^0_{ij} \frac{\partial^2}{\partial x_i \partial x_j} a_5 = 0 , \tag{11}$$

$$\frac{\partial}{\partial t} a_\mu - \frac{1}{2} A^{1/2} \lambda^0_{\mu j \alpha} \frac{\partial}{\partial x_j} a_\alpha - \frac{1}{\gamma_1} \delta^{0\,\perp}_{\mu k} \kappa^0_{kj\nu l} \frac{\partial^2}{\partial x_j \partial x_l} a_\nu = 0 , \tag{12}$$

where the Latin subindices run over the range $i,j,k,l,m = 1,2,3,4,5,6,7$ and the quantities B, $\delta^{0\,\perp}_{ik}$, $v^0_{ij\beta l}$, κ^0_{ij}, λ^0_{kji}, $K^0_{kj\mu l}$, were also defined in the Appendix. It is important to point out that as shown by Eqs. (10) and (12), the velocity and director fields are coupled, that is, the translational motions are coupled to inner, orientational motions of the molecules. In most cases the flow disturbs the alignment and conversely, a change in the alignment (e.g., by the application of an external field) will induce a flow in the nematic. However, as will be discussed later on, the characteristic time scales of these processes are quite different and the response of the director to a superimposed flow is not instantaneous.

In matrix form Eqs. (9)–(12) can be rewritten in the form

$$\frac{\partial}{\partial t} a_i(\vec{r},t) = -\int G_{ij}(\vec{r},\vec{r}') a_j(\vec{r}',t)\, \mathrm{d}^3 r' , \tag{13}$$

where the matrix G_{ij} is symmetric

$$G_{ij}(\vec{r},\vec{r}') = \begin{pmatrix} 0 & G_{1\alpha} & 0 & 0 \\ G_{\alpha 1} & G_{\alpha\beta} & G_{\alpha 5} & G_{\alpha\mu} \\ 0 & G_{5\alpha} & G_{55} & 0 \\ 0 & G_{\mu\alpha} & 0 & G_{\mu\nu} \end{pmatrix} \tag{14}$$

with

$$G_{1\alpha} = G_{\alpha 1} = A^{1/2} \frac{\partial}{\partial x_\alpha} \delta(\vec{r} - \vec{r}') , \tag{15}$$

$$G_{\alpha\beta} = \frac{1}{\rho_0} v^0_{\alpha j \beta l} \frac{\partial^2}{\partial x_j \partial x'_l} \delta(\vec{r} - \vec{r}') , \tag{16}$$

$$G_{\alpha 5} = G_{5\alpha} = \frac{B}{\rho_0} \left(\frac{T_0}{C}\right)^{1/2} \frac{\partial}{\partial x_\alpha} \delta(\vec{r} - \vec{r}') , \tag{17}$$

$$G_{55} = \frac{1}{\rho_0 C} \kappa^0_{ij} \frac{\partial^2}{\partial x_i \partial x'_j} \delta(\vec{r} - \vec{r}') , \tag{18}$$

$$G_{\alpha\mu} = -\frac{1}{2\rho_0 A^{1/2}} \lambda^0_{kj\alpha} K^0_{ks\mu r} \frac{\partial^3}{\partial x_j \partial x_s \partial x'_r} \delta(\vec{r} - \vec{r}') , \tag{19}$$

$$G_{\mu\alpha} = -\frac{1}{2} A^{1/2} \lambda^0_{\mu j \alpha} \frac{\partial}{\partial x_j} \delta(\vec{r} - \vec{r}') , \tag{20}$$

$$G_{\mu\nu} = \frac{1}{\gamma_1} \delta^{0\,\perp}_{\mu k} K^0_{kj\nu l} \frac{\partial^2}{\partial x_j \partial x'_l} \delta(\vec{r} - \vec{r}') \tag{21}$$

and where $\delta(\vec{r} - \vec{r}')$ is Dirac's delta function. Note that G_{ij} may be decomposed in the form $G_{ij}(\vec{r},\vec{r}') = S_{ij}(\vec{r},\vec{r}') + A_{ij}(\vec{r},\vec{r}')$ with

$$S_{ij}(\vec{r},\vec{r}') = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & S_{\alpha\beta} & 0 & S_{\alpha\mu} \\ 0 & 0 & S_{55} & 0 \\ 0 & S_{\mu\alpha} & 0 & S_{\mu\nu} \end{pmatrix} , \tag{22}$$

where

$$S_{\alpha\beta} = \frac{1}{\rho_0} v^0_{\alpha j \beta l} \frac{\partial^2}{\partial x_j \partial x'_l} \delta(\vec{r} - \vec{r}') , \tag{23}$$

$$S_{55} = \frac{1}{\rho_0 C} \kappa^0_{ij} \frac{\partial^2}{\partial x_i \partial x'_j} \delta(\vec{r} - \vec{r}') , \tag{24}$$

$$S_{\alpha\mu} = -\frac{1}{4}\left[\frac{1}{\rho_0 A^{1/2}}\lambda^0_{kj\alpha}K^0_{ks\mu r}\frac{\partial^3}{\partial x_j \partial x_s \partial x'_r} - A^{1/2}\lambda^0_{\mu j\alpha}\frac{\partial}{\partial x_j}\right]\delta(\vec{r}-\vec{r}') , \quad (25)$$

$$S_{\mu\alpha} = -\frac{1}{4}\left[A^{1/2}\lambda^0_{\mu j\alpha}\frac{\partial}{\partial x_j} - \frac{1}{\rho_0 A^{1/2}}\lambda^0_{kj\alpha}K^0_{ks\mu r}\frac{\partial^3}{\partial x_j \partial x_s \partial x'_r}\right]\delta(\vec{r}-\vec{r}') , \quad (26)$$

$$S_{\mu\nu} = \frac{1}{\gamma_1}\delta^{0\,\perp}_{\mu k}K^0_{kj\nu l}\frac{\partial^2}{\partial x_j \partial x'_l}\delta(\vec{r}-\vec{r}') \quad (27)$$

and

$$A_{ij}(\vec{r},\vec{r}') = \begin{pmatrix} 0 & A_{1\alpha} & 0 & 0 \\ A_{\alpha 1} & 0 & A_{\alpha 5} & A_{\alpha\mu} \\ 0 & A_{5\alpha} & 0 & 0 \\ 0 & A_{\mu\alpha} & 0 & 0 \end{pmatrix} \quad (28)$$

with

$$A_{1\alpha} = A_{\alpha 1} = A^{1/2}\frac{\partial}{\partial x_\alpha}\delta(\vec{r}-\vec{r}') , \quad (29)$$

$$A_{\alpha 5} = A_{5\alpha} = \frac{B}{\rho_0}\left(\frac{T_0}{C}\right)^{1/2}\frac{\partial}{\partial x_\alpha}\delta(\vec{r}-\vec{r}') , \quad (30)$$

$$A_{\alpha\mu} = -\frac{1}{4}\left[\frac{1}{\rho_0 A^{1/2}}\lambda^0_{kj\alpha}K^0_{ks\mu r}\frac{\partial^3}{\partial x_j \partial x_s \partial x'_r} + A^{1/2}\lambda^0_{\mu j\alpha}\frac{\partial}{\partial x_j}\right]\delta(\vec{r}-\vec{r}') , \quad (31)$$

$$A_{\mu\alpha} = -\frac{1}{4}\left[A^{1/2}\lambda^0_{\mu j\alpha}\frac{\partial}{\partial x_j} + \frac{1}{\rho_0 A^{1/2}}\lambda^0_{kj\alpha}K^0_{ks\mu r}\frac{\partial^3}{\partial x_j \partial x_s \partial x'_r}\right]\delta(\vec{r}-\vec{r}') . \quad (32)$$

The matrices A_{ij} and S_{ij} are, respectively, anti-symmetric and symmetric with respect to interchange of discrete and continuous indices $\vec{r}$ and $\vec{r}'$, i.e.,

$$A_{ij}(\vec{r},\vec{r}') = -A_{ji}(\vec{r}',\vec{r}) , \quad (33)$$

$$S_{ij}(\vec{r},\vec{r}') = S_{ji}(\vec{r}',\vec{r}) \quad (34)$$

provided that discrete indices, i, j, are summed over and $\vec{r}$, $\vec{r}'$are considered as continuous indices whose summation is indicated by the integral in Eq. (13). Thus, the hydrodynamic set of linearized equations (9)–(12) may be rewritten in the general form

$$\frac{\partial}{\partial t}a_i(\vec{r},t) = -\int A_{ij}(\vec{r},\vec{r}')a_j(\vec{r}',t)\,\mathrm{d}^3r' - \int S_{ij}(\vec{r},\vec{r}')a_j(\vec{r}',t)\,\mathrm{d}^3r' . \quad (35)$$

Before introducing fluctuating terms into this equation in the following subsection, we first derive an expression for the time rate of change of the total entropy in terms of the variables $\{a_1, a_\alpha, a_5, a_\mu\}$. The total entropy is given by

$$S(t) \equiv \int_V \rho\sigma \, \mathrm{d}^3 r \,, \tag{36}$$

where σ is the entropy per unit mass. From Eqs. (9)–(12), its time rate of change in terms of the variables Eqs. (5)–(8) is computed to be

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} S(t) = \frac{1}{T_0} \int \Bigg\{ & a_5 \left(\frac{A}{\rho_0 C} \kappa^0_{ij} \frac{\partial}{\partial x_i} \frac{\partial}{\partial x'_j} \right) a_5 + a_\alpha \left(\frac{A}{\rho_0} \nu^0_{\alpha j \beta l} \frac{\partial}{\partial x_j} \frac{\partial}{\partial x'_l} \right) a_\beta \\ & + a_\mu \left[\frac{1}{\gamma_1 \rho_0} \delta^0_{ij} \perp \left(K^0_{is\mu l} \frac{\partial^2}{\partial x_s \partial x'_l} \right) \left(K^0_{jm\nu l} \frac{\partial^2}{\partial x_m \partial x'_l} \right) \right] a_\nu \Bigg\} \mathrm{d}V \,. \end{aligned} \tag{37}$$

On the other hand, near equilibrium the entropy has a quadratic representation

$$\mathscr{S}(t) = \mathscr{S}^o - \frac{1}{2} k_B \int \int a_i(\vec{r}, t) E_{ij}(\vec{r}, \vec{r}') a_j(\vec{r}', t) \, \mathrm{d}^3 r \, \mathrm{d}^3 r' \tag{38}$$

and the time rate $\frac{\mathrm{d}}{\mathrm{d}t} S(t)$ can be recasted in the form

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t} S(t) = \frac{1}{2} k_B \int \int \int & a_i(\vec{r}, t) [E_{ik}(\vec{r}, \vec{r}'') S_{kj}(\vec{r}'', \vec{r}') \\ & + S_{ik}(\vec{r}, \vec{r}'') E_{kj}(\vec{r}'', \vec{r}') + E_{ik}(\vec{r}, \vec{r}'') A_{kj}(\vec{r}'', \vec{r}') \\ & - A_{ik}(\vec{r}, \vec{r}'') E_{kj}(\vec{r}'', \vec{r}')] a_j(\vec{r}', t) \, \mathrm{d}^3 r \, \mathrm{d}^3 r' \, \mathrm{d}^3 r'' \,. \end{aligned} \tag{39}$$

Using Eqs. (23), (29) and equating (35) with (39), we arrive at the following form for the inverse of the entropy matrix E_{ij}

$$\begin{aligned} E^{-1}_{ij}(\vec{r}, \vec{r}') = \frac{k_B T_0}{A} & \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \delta_{\alpha\beta} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \rho_0 A (K^0_{klij} \frac{\partial^2}{\partial x_k x^0_l})^{-1} \end{pmatrix} \\ & \times \delta(\vec{r} - \vec{r}'), \quad i, j = 1, \ldots, 8 \end{aligned} \tag{40}$$

with $\alpha, \beta = 2, 3, 4$ and $\mu, \nu = 6, 7, 8$.

We shall now determine the correlation matrix $\overleftrightarrow{Q}$. To this end recall that the stationary property of the underlying stochastic process requires that [3],

$$2Q_{ij}(\vec{r}, \vec{r}') = \int [G_{ik}(\vec{r}, \vec{r}'') E^{-1}_{kj}(\vec{r}'', \vec{r}') + E^{-1}_{ik}(\vec{r}, \vec{r}'') G^\dagger_{kj}(\vec{r}'', \vec{r}')] \, \mathrm{d}^3 r'' \,. \tag{41}$$

Thus, upon substitution of Eqs. (22), (23) and (29) we arrive at

$$Q_{ij}(\vec{r},\vec{r}\,') = \frac{k_B T_0}{A} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & \frac{1}{\rho_0} v_{ikjl} \frac{\partial^2}{\partial x_j \partial x'_l} & 0 & 0 \\ 0 & 0 & \frac{1}{\rho_0 C} \kappa^0_{ij} \frac{\partial^2}{\partial x_i \partial x^0_j} & 0 \\ 0 & 0 & 0 & \rho_0 A \frac{1}{\gamma_1} \delta^{0\ \perp}_{ij} \end{pmatrix} \delta(\vec{r}-\vec{r}\,') , \tag{42}$$

where again, $\alpha, \beta = 2, 3, 4$ and $\mu, v = 6, 7, 8$.

2.2. Fluctuation–dissipation relations

The introduction of an additive fluctuating force $F_i(\vec{r}, t)$ into the hydrodynamic equations Eq. (35) transforms them into a set of stochastic equations of the Langevin form where $\vec{F}$ is a set of stationary, random Gaussian forces with

$$\langle \vec{F}(\vec{r}, t) \rangle = 0 , \tag{43}$$

$$\langle F_i(\vec{r}, t) F^{\dagger}_j(\vec{r}\,', t') \rangle = 2Q_{ij}(\vec{r}, \vec{r}\,') \delta(t - t') , \tag{44}$$

where the superscript † denotes a Hermitian conjugate. Note, however, that since $Q_{11} = 0$ there can be no fluctuating force added to the continuity equation. Also, all the elements of the correlation matrix $\overleftrightarrow{Q}$ contain partial derivatives, except $Q_{\mu v} = (\rho_0 T_0/\gamma_1)\delta^{0\ \perp}_{\mu v}$. To eliminate these partial derivatives it is convenient to work with the random quantities $\Sigma_{\alpha\beta}(\vec{r}, t)$, $\pi_l(\vec{r}, t)$ and $\Upsilon_\mu(\vec{r}, t)$, defined in terms of the stochastic forces by

$$F_\alpha(\vec{r}, t) \equiv \left(\frac{1}{\rho_0 A} \right)^{1/2} \frac{\partial}{\partial x_\beta} \Sigma_{\alpha\beta}(\vec{r}, t) , \tag{45}$$

$$F_5(\vec{r}, t) \equiv \left(\frac{1}{\rho_0 T_0 A C} \right)^{1/2} \frac{\partial}{\partial x_l} g_l(\vec{r}, t) , \tag{46}$$

$$F_\mu(\vec{r}, t) \equiv (\rho_0)^{1/2} \Upsilon_\mu(\vec{r}, t) . \tag{47}$$

Substitution of Eq. (42) into Eq. (44) yields

$$\begin{aligned} \langle F_\alpha(\vec{r}, t) F_\beta(\vec{r}\,', t') \rangle &= 2Q_{\alpha\beta}(\vec{r}, \vec{r}\,') \delta(t - t') \\ &= \frac{2k_B T_0}{A \rho_0} v^0_{\alpha\beta jl} \frac{\partial^2}{\partial x_j \partial x'_l} \delta(\vec{r} - \vec{r}\,') \delta(t - t') . \end{aligned} \tag{48}$$

On the other hand, from Eq. (45) it follows that

$$\langle F_\alpha(\vec{r}, t) F_\beta(\vec{r}\,', t') \rangle = \frac{1}{\rho_0 A} \frac{\partial^2}{\partial x_\gamma \partial x'_\varepsilon} \langle \Sigma_{\alpha\gamma}(\vec{r}, t) \Sigma_{\beta\varepsilon}(\vec{r}\,', t') \rangle , \tag{49}$$

so that equating Eqs. (48) and (49) gives

$$\langle \Sigma_{\alpha j}(\vec{r},t)\Sigma_{\beta l}(\vec{r}',t')\rangle = 2k_B T_0 \nu^0_{\alpha\beta jl}\delta(\vec{r}-\vec{r}')\delta(t-t') . \tag{50}$$

Similarly, substitution of Q_{55} from Eq. (42) into Eq. (44) leads to

$$\begin{aligned}\langle F_5(\vec{r},t)F_5(\vec{r}',t')\rangle &= 2Q_{55}\delta(\vec{r}-\vec{r}')\delta(t-t')\\ &= \frac{2k_B T_0}{A}\frac{1}{\rho_0 C}\kappa^0_{ij}\frac{\partial^2}{\partial x_i \partial x'_j}\delta(\vec{r}-\vec{r}')\delta(t-t')\end{aligned} \tag{51}$$

while Eq. (46) implies

$$\langle F_5(\vec{r},t)F_5(\vec{r}',t')\rangle = \left(\frac{1}{\rho_0 T_0 A C}\right)\frac{\partial^2}{\partial x_i \partial x'_j}\langle g_i(\vec{r},t)g_j(\vec{r}',t')\rangle . \tag{52}$$

Thus, comparison of Eqs. (51) and (52) gives

$$\langle g_i(\vec{r},t)g_j(\vec{r}',t')\rangle = 2k_B T_0^2 \kappa^0_{ij}\delta(\vec{r}-\vec{r}')\delta(t-t') . \tag{53}$$

In a similar fashion we arrive at

$$\langle \Upsilon_\mu(\vec{r},t)\Upsilon_\nu(\vec{r}',t')\rangle = 2k_B T_0 \frac{1}{\gamma_1}\delta^{0\ \perp}_{\mu\nu}\delta(\vec{r}-\vec{r}')\delta(t-t') . \tag{54}$$

Eqs. (50), (53) and (54) relate the stochastic quantities $\Sigma_{\alpha j}(\vec{r},t)$, $g_i(\vec{r},t)$, $\Upsilon_\mu(\vec{r},t)$ with the dissipative quantities $\nu^0_{\alpha\beta jl}$, κ^0_{ij}, $\frac{1}{\gamma_1}\delta^{0\ \perp}_{\mu\nu}$, and are, therefore, the fluctuation–dissipation relations for a thermotropic nematic.

Thus, the introduction of a fluctuating force $F_i(\vec{r},t)$ which are random forces described by a white Gaussian processes with zero mean $\langle F_i(\vec{r},t)\rangle = 0$ and autocorrelations given by Eqs. (48), (51) and (52), yields the following hydrodynamic set of linearized stochastic nematodynamic equations for a thermotropic nematic:

$$\begin{aligned}\frac{\partial}{\partial t}a_i(\vec{r},t) = &-\int S_{ij}(\vec{r},\vec{r}')a_j(\vec{r}',t)\,\mathrm{d}^3r' - \int A_{ij}(\vec{r},\vec{r}')a_j(\vec{r}',t)\,\mathrm{d}^3r'\\ &+ F_i(\vec{r},t) .\end{aligned} \tag{55}$$

This form is identical to that obtained by Fox and Uhlenbeck showing that, as expected, the general theory by Fox and Uhlenbeck also describes the dynamics of thermal fluctuations of a thermotropic nematic liquid crystal. However, although expected, the explicit derivation of this result is certainly not a trivial exercise.

Furthermore, from the theory of Gaussian stochastic processes it is also possible to give the explicit form of the Fokker–Planck equation for the conditioned two time distribution function $P_2(a_0,t_0;a,t)$. This equation is given by Eq. (I.2.39) in Ref. [3]

$$\frac{\partial}{\partial t}P_2(\vec{a},t;\vec{a}_0) = \frac{\partial}{\partial a_i}(G_{ij}a_j P_2) + \frac{1}{2}\frac{\partial}{\partial a_i}\left(Q_{ij}\frac{\partial}{\partial a_j}P_2\right) \tag{56}$$

with the initial condition

$$P_2(\vec{a},t;\vec{a}_0) = \delta(\vec{a}-\vec{a}_0) . \tag{57}$$

Clearly, for the present model from Eqs. (14)–(21) and (42), the Fokker–Planck equation is completely determined.

3. Transverse correlation functions

Due to the complexity of Eqs. (55) and for the purpose of calculating the correlation functions explicitly, from now on we restrict our analysis to the transverse hydrodynamic variables $\{a_2(\vec{r},t),\ a_6(\vec{r},t)\}$. The Fourier transform of an arbitrary field $A(\vec{r},t)$ is defined by

$$\widetilde{A}(\vec{k},\omega) \equiv \int_{-\infty}^{\infty} A(\vec{r},t)\mathrm{e}^{-\mathrm{i}(\vec{k}\cdot\vec{r}-\omega t)}\,\mathrm{d}\vec{r}\,\mathrm{d}t \tag{58}$$

with

$$A(\vec{r},t) = \frac{1}{(2\pi)^4}\int_{-\infty}^{\infty} \widetilde{A}(\vec{k},\omega)\mathrm{e}^{\mathrm{i}(\vec{k}\cdot\vec{r}-\omega t)}\,\mathrm{d}\vec{k}\,\mathrm{d}\omega\,. \tag{59}$$

Following Ref. [31], we know that those terms in the equations of motion for $a_2(\vec{r},t) \equiv \left(\frac{\rho_0}{A}\right)^{1/2}\delta v_x(\vec{r},t)$ and $a_6(\vec{r},t) \equiv \rho_0^{1/2}\delta n_x(\vec{r},t)$ that are proportional to z and z^2 will not contribute to the correlation functions up to first order in the external gradient. Thus, neglecting these terms in the equations of motion (10) and (12), in Fourier space they may be rewritten in matrix form as

$$\begin{bmatrix} \widetilde{L}_{11} & \widetilde{L}_{12} \\ \widetilde{L}_{21} & \widetilde{L}_{22} \end{bmatrix} \begin{bmatrix} \widetilde{a}_2(\vec{k},\omega) \\ \widetilde{a}_6(\vec{k},\omega) \end{bmatrix} = \begin{bmatrix} \widetilde{f}_2(\vec{k},\omega) \\ \widetilde{f}_6(\vec{k},\omega) \end{bmatrix} \tag{60}$$

with

$$\widetilde{L}_{11}(\vec{k},\omega) = \left(\frac{A}{\rho_0}\right)^{1/2}(-\mathrm{i}\omega + \Delta_{11} + \mathrm{i}\gamma b_1 k_y)\,, \tag{61}$$

$$\widetilde{L}_{12}(\vec{k},\omega) = -\mathrm{i}\rho_0^{-1/2}\Delta_{12}\,, \tag{62}$$

$$\widetilde{L}_{21}(\vec{k},\omega) = -\mathrm{i}\left(\frac{A}{\rho_0}\right)^{1/2}\Delta_{21}\,, \tag{63}$$

$$\widetilde{L}_{22}(\vec{k},\omega) = \rho_0^{-1/2}(-\mathrm{i}\omega + \Delta_{22} + \mathrm{i}\gamma b_1 k_y)\,, \tag{64}$$

$$\widetilde{f}_2(\vec{k},\omega) = -\frac{\mathrm{i}}{\rho_0}k_n\widetilde{\Sigma}_{xn}(\vec{k},\omega), n = y, z\,, \tag{65}$$

$$\widetilde{f}_6(\vec{k},\omega) = -\widetilde{\Upsilon}_x(\vec{k},\omega) \tag{66}$$

and where the following abbreviations have been used, $\Delta_{11} \equiv \frac{v_2k_y^2+v_3k_z^2}{\rho_0}$, $\Delta_{12} \equiv \frac{\lambda+1}{2\rho_0}(K_2k_y^2 + K_3k_z^2)k_z$, $\Delta_{21} \equiv \frac{\lambda+1}{2}k_z$, $\Delta_{22} \equiv \frac{K_2k_y^2+K_3k_z^2}{\gamma_1}$. The random forces autocorrelations $\langle \widetilde{f}_2(\vec{k},\omega)\widetilde{f}_2(\vec{k}',\omega')\rangle$ and $\langle \widetilde{f}_6(\vec{k},\omega)\widetilde{f}_6(\vec{k}',\omega')\rangle$are obtained from Eqs. (50), (54) and (58)

$$\langle \widetilde{f}_2(\vec{k},\omega)\widetilde{f}_2(\vec{k}',\omega')\rangle = -\frac{2k_B T_0 (2\pi)^4}{\rho_0^2} k_j k_l' v^0_{xxjl}\delta(\vec{k}+\vec{k}')\delta(\omega+\omega') , \tag{67}$$

$$\langle \widetilde{f}_6(\vec{k},\omega)\widetilde{f}_6(\vec{k}',\omega')\rangle = \frac{2k_B T_0 (2\pi)^4}{\gamma_1}\delta(\vec{k}+\vec{k}')\delta(\omega+\omega') . \tag{68}$$

Here $\delta(\vec{k}+\vec{k}')$ and $\delta(\omega+\omega')$ denote the corresponding Dirac's delta functions.

The matrix $\overleftrightarrow{L}$ may be simplified by noting that for a typical thermotropic nematic the material parameters have the values $v_2 \sim v_3 \sim \gamma_1 \sim 10^{-1}$ poise, $\rho_0 \sim 1\,\mathrm{g/cm^3}$, $K_2 \sim K_3 \sim 10^{-7}$ dyn, $\lambda \sim 1$ [43], so that the following inequalities hold:

$$\frac{\Delta_{22}}{\Delta_{11}} \sim \frac{K\rho}{v^2} \ll 1 , \tag{69}$$

$$\frac{\Delta_{12}\Delta_{21}}{\Delta_{11}^2} \sim \left(\frac{K\rho}{v^2}\right)^2 \ll 1 . \tag{70}$$

Using these results the inverse matrix

$$\overleftrightarrow{N}(\vec{k},\omega) \equiv \overleftrightarrow{L}^{-1}(\vec{k},\omega) = \frac{1}{\det \overleftrightarrow{L}}\begin{bmatrix} \widetilde{L}_{22} & -\widetilde{L}_{12} \\ -\widetilde{L}_{21} & \widetilde{L}_{11}\end{bmatrix} \tag{71}$$

may be simplified as will be shown below. To this end first note that it can be shown that

$$\widetilde{D} \equiv \frac{\rho_0}{\sqrt{A}} \det \overleftrightarrow{L} = (\mathrm{i}\omega+\omega_1)(\mathrm{i}\omega+\omega_2) , \tag{72}$$

where ω_1 and ω_2 are the two transverse modes. The solution of $\det \overleftrightarrow{L} = 0$ leads to the explicit forms of the equilibrium and nonequilibrium parts of these modes

$$\begin{aligned}\omega_1 &\equiv \omega_1^{eq} + \omega_1^{neq} \\ &= -\frac{\mathrm{i}}{\rho_0}(v_2 k_y^2 + v_3 k_z^2) + \gamma b_1 k_y ,\end{aligned} \tag{73}$$

where the term linear in γ, ω_1^{neq}, corresponds to the nonequilibrium part of the mode. Similarly,

$$\begin{aligned}\omega_2 &= \omega_2^{eq} + \omega_2^{neq} \\ &= -\mathrm{i}(K_2 k_y^2 + K_3 k_z^2)\left[\frac{1}{\gamma_1} + \frac{(\lambda+1)^2}{4}\frac{k_z^2}{v_2 k_y^2 + v_3 k_z^2}\right] + \gamma b_1 k_y ,\end{aligned} \tag{74}$$

where $\omega_2^{neq} \equiv \gamma b_1 k_y$. The mode $\omega_1 \sim -\frac{\mathrm{i}vk^2}{\rho}$ is associated with the diffusive shear velocity $\widetilde{a}_2(\vec{k},\omega)$, whereas $\omega_2 \sim -\frac{\mathrm{i}Kk^2}{v}$ describes the relaxation mode of the transverse director component. Furthermore, since for a typical thermotropic nematic [43]

$$\frac{\omega_2}{\omega_1} = \frac{\Delta_{11}\Delta_{22} + \Delta_{12}\Delta_{21}}{(\Delta_{11})^2} \sim 10^{-5} \ll 1 \tag{75}$$

this implies that the relaxation time of the reorientation, $\widetilde{a}_6(\vec{k},\omega)$, induced by the flow is much larger ($\sim 10^5$) than the characteristic time associated with the velocity, $\widetilde{a}_2(\vec{k},\omega)$. Thus, ω_1 represents a fast mode and ω_2 a slow mode and it is reasonable to expect that the director profile does not distort immediately when a flow is superimposed, even in the linear regime considered here.

From Eqs. (69) and (70) it follows that up to first order in the external gradient γ

$$\widetilde{D} = \widetilde{D}^{eq} - \gamma b_1 k_y (2\omega - \mathrm{i}\Delta_{11}) , \tag{76}$$

where

$$\widetilde{D}^{eq} = (\omega_1^{eq} + \mathrm{i}\omega)(\omega_2^{eq} + \mathrm{i}\omega) \tag{77}$$

and then

$$\widetilde{D}^{-1} = (\widetilde{D}^{eq})^{-1}\left[1 + \gamma \frac{b_1 k_y}{\widetilde{D}^{eq}}(2\omega - \mathrm{i}\Delta_{11})\right] . \tag{78}$$

Thus, the solution of Eq. (60) reads

$$\widetilde{a}_2(\vec{k},\omega) = \widetilde{a}_2^{eq}(\vec{k},\omega) + \widetilde{a}_2^{neq}(\vec{k},\omega) \tag{79}$$

with

$$\widetilde{a}_2^{eq}(\vec{k},\omega) \equiv \frac{1}{\widetilde{D}^{eq}}\left(\frac{\rho_0}{A}\right)^{1/2}[(\mathrm{i}\omega - \Delta_{22})\widetilde{f}_2 - \mathrm{i}\Delta_{12}\widetilde{f}_6] , \tag{80}$$

$$\widetilde{a}_2^{neq}(\vec{k},\omega) = \gamma \frac{b_1 k_y}{\widetilde{D}^{eq}}\left(\frac{\rho_0}{A}\right)^{1/2}\left[-\mathrm{i}\widetilde{f}_2 + \left(\frac{A}{\rho_0}\right)^{1/2}(2\omega - \mathrm{i}\Delta_{11})a_2^{eq}(\vec{k},\omega)\right] \tag{81}$$

and

$$\widetilde{a}_6(\vec{k},\omega) = \widetilde{a}_6^{eq}(\vec{k},\omega)^{eq} + \widetilde{a}_6^{neq}(\vec{k},\omega) \tag{82}$$

with

$$\widetilde{a}_6^{eq}(\vec{k},\omega) \equiv -\frac{\rho_0^{1/2}}{\widetilde{D}^{eq}}[\mathrm{i}\Delta_{21}\widetilde{f}_2 + (-\mathrm{i}\omega + \Delta_{11})\widetilde{f}_6] , \tag{83}$$

$$\widetilde{a}_6^{neq}(\vec{k},\omega) \equiv \gamma \frac{\rho_0^{1/2} b_1 k_y}{\widetilde{D}^{eq}}[\mathrm{i}\widetilde{f}_6 + (2\omega - \mathrm{i}\Delta_{11})\rho_0^{-1/2}\widetilde{a}_6^{eq}] . \tag{84}$$

The right-hand sides of Eqs. (79) and (82) contain all the information about the nonequilibrium effects produced by the external pressure gradient through the presence of the dimensionless parameter γ, which measures the degree of departure from the stationary state.

From these solutions the equilibrium and nonequilibrium parts of the correlation matrix for the transverse modes, namely,

$$\overleftrightarrow{\chi}(\vec{k},\omega) \equiv \overleftrightarrow{\chi}^{eq}(\vec{k},\omega) + \overleftrightarrow{\chi}^{neq}(\vec{k},\omega)\,, \tag{85}$$

can now be calculated explicitly. Using Eqs. (80), (83), the fluctuation–dissipation relations (50), (53), (54), inequalities (69), (70) and performing a partial fractions expansion, we arrive at the following expression for the equilibrium part of the correlation matrix:

$$\widetilde{\chi}^{eq}_{ij}(\vec{k},\omega) = \begin{bmatrix} \widetilde{\chi}^{eq}_{11} & \widetilde{\chi}^{eq}_{12} \\ \widetilde{\chi}^{eq}_{21} & \widetilde{\chi}^{eq}_{22} \end{bmatrix} \tag{86}$$

with

$$\begin{aligned}\widetilde{\chi}^{eq}_{11}(\vec{k},\omega) &\equiv \langle \widetilde{a}^{eq}_2(\vec{k},\omega)\widetilde{a}^{eq}_2(-\vec{k},-\omega)\rangle \\ &= \frac{2(2\pi)^4\delta^4(0)}{A} k_B T \frac{\Delta_{11}}{\omega^2 + \mid \omega^{eq}_1 \mid^2}\,,\end{aligned} \tag{87}$$

$$\begin{aligned}\widetilde{\chi}^{eq}_{22}(\vec{k},\omega) &\equiv \langle \widetilde{a}^{eq}_6(\vec{k},\omega)\widetilde{a}^{eq}_6(-\vec{k},-\omega)\rangle \\ &= \frac{2(2\pi)^4\delta^4(0)\rho_0}{\gamma_1} k_B T \left[\frac{1+\alpha}{\omega^2 + \mid \omega^{eq}_2 \mid^2} - \frac{\alpha}{\omega^2 + \mid \omega^{eq}_1 \mid^2}\right],\end{aligned} \tag{88}$$

$$\begin{aligned}\widetilde{\chi}^{eq}_{12}(\vec{k},\omega) &\equiv \langle \widetilde{a}^{eq}_2(\vec{k},\omega)\widetilde{a}^{eq}_6(-\vec{k},-\omega)\rangle \\ &= \frac{2(2\pi)^4\delta^4(0)}{A^{1/2}} k_B T \omega \frac{\Delta_{21}}{\Delta_{11}} \left[\frac{1}{\omega^2 + \mid \omega^{eq}_2 \mid^2} - \frac{1}{\omega^2 + \mid \omega^{eq}_1 \mid^2}\right],\end{aligned} \tag{89}$$

$$\widetilde{\chi}^{eq}_{21}(\vec{k},\omega) \equiv \langle \widetilde{a}^{eq}_6(\vec{k},\omega)\widetilde{a}^{eq}_2(-\vec{k},-\omega)\rangle = \widetilde{\chi}^{eq}_{12}(\vec{k},\omega)\,, \tag{90}$$

where $\alpha(\varphi) \equiv \frac{\gamma_1(\lambda+1)^2}{4}\frac{1}{v_2\tan^2\varphi + v_3}$.

In a similar fashion the nonequilibrium part of the correlation matrix,

$$\widetilde{\chi}^{neq}_{ij}(\vec{k},\omega) \equiv \begin{bmatrix} \widetilde{\chi}^{neq}_{11} & \widetilde{\chi}^{neq}_{12} \\ \widetilde{\chi}^{neq}_{21} & \widetilde{\chi}^{neq}_{22} \end{bmatrix} \tag{91}$$

turns out to be

$$\begin{aligned}\widetilde{\chi}^{neq}_{11}(\vec{k},\omega) &\equiv \langle \widetilde{a}^{eq}_2(\vec{k},\omega)\widetilde{a}^{neq}_2(-\vec{k},-\omega)\rangle + \langle \widetilde{a}^{neq}_2(\vec{k},\omega)\widetilde{a}^{eq}_2(-\vec{k},-\omega)\rangle \\ &= \gamma b_1 k_y \frac{4(2\pi)^4\delta^4(0)}{A} k_B T \frac{\omega\Delta_{11}}{\omega^2 + \mid \omega^{eq}_1 \mid^2}\,,\end{aligned} \tag{92}$$

$$
\begin{aligned}
\widetilde{\chi}_{22}^{neq}(\vec{k},\omega) &\equiv \langle \widetilde{a}_6^{eq}(\vec{k},\omega)\widetilde{a}_6^{neq}(-\vec{k},-\omega)\rangle + \langle \widetilde{a}_6^{neq}(\vec{k},\omega)\widetilde{a}_6^{eq}(-\vec{k},-\omega)\rangle \\
&= \gamma b_1 k_y \frac{4(2\pi)^4\delta^4(0)}{\gamma_1}\rho_0 k_B T \left[\frac{4+5\alpha+(1+2\alpha)\Delta_{11}^{-2}\omega^2}{(\omega^2+\mid\omega_1^{eq}\mid^2)^2}\right. \\
&\quad \left. -\frac{1+\alpha+(1+2\alpha)\Delta_{11}^{-2}\omega^2}{(\omega^2+\mid\omega_2^{eq}\mid^2)^2}\right], \qquad (93)
\end{aligned}
$$

$$
\begin{aligned}
\widetilde{\chi}_{12}^{neq}(\vec{k},\omega) &\equiv \langle \widetilde{a}_2^{eq}(\vec{k},\omega)\widetilde{a}_6^{neq}(-\vec{k},-\omega)\rangle + \langle \widetilde{a}_2^{neq}(\vec{k},\omega)\widetilde{a}_6^{eq}(-\vec{k},-\omega)\rangle \\
&= \gamma b_1 k_y \frac{4(2\pi)^4\delta^4(0)}{A^{1/2}} k_B T \left[\frac{1-\frac{\omega^2}{(\Delta_{11})^2}}{(\omega^2+\mid\omega_1^{eq}\mid^2)^2} + \frac{\frac{\omega^2}{(\Delta_{11})^2}-\left(\frac{\Delta_{22}}{\Delta_{11}}\right)^2}{(\omega^2+\mid\omega_2^{eq}\mid^2)^2}\right], \qquad (94)
\end{aligned}
$$

where

$$
\begin{aligned}
\widetilde{\chi}_{21}^{neq}(\vec{k},\omega) &\equiv \langle \widetilde{a}_6^{eq}(\vec{k},\omega)\delta\widetilde{a}_2^{neq}(-\vec{k},-\omega)\rangle + \langle \widetilde{a}_6^{neq}(\vec{k},\omega)\widetilde{a}_2^{eq}(-\vec{k},-\omega)\rangle \\
&= \widetilde{\chi}_{12}^{neq}(\vec{k},\omega)\,. \qquad (95)
\end{aligned}
$$

In the above equations $\delta^4(0)$ is obtained by evaluating $\delta(\vec{k}+\vec{k}')$ and $\delta(\omega+\omega')$ at $\vec{k}=-\vec{k}'$ and $\omega=-\omega'$, respectively.

4. Light scattering

At a large distance R, the scattered electric field E_s of a plane wave incident on a nonmagnetic, nonconducting and nonabsorbent medium with an average electric permittivity ε_0, is proportional to the Fourier transform of its dielectric tensor fluctuations $\delta\varepsilon_{if}(\vec{k},t)$ [44],

$$
E_s(\vec{R},t) = \frac{\vec{k}_f^2 E_0 e^{i(\vec{k}_f\cdot\vec{R}-\omega_i t)}}{4\pi\varepsilon_0 R}\delta\varepsilon_{if}(\vec{k},t)\,, \qquad (96)
$$

where ω_i is the angular frequency of the incident field and the difference between the wave vectors of the incident and outgoing waves, $\vec{k}\equiv\vec{k}_i-\vec{k}_f$, is the scattering vector. The plane $(\vec{k}_i,\vec{k}_f)$ is the scattering plane and the angle between $\vec{k}_i$ and $\vec{k}_f$ is the (internal) scattering angle θ. $\widehat{i}$ and $\widehat{f}$ are the initial and final directions of the polarizations of the incident and scattered light and φ is the angle between $\vec{k}$ and the z axis (see Fig. 1). Note that from Fig. 1 it follows that $k=2k_i\sin(\theta/2)$ and $\varphi=\theta/2$. From here on, we shall denote the shifts in frequency by $\omega\equiv\omega_f-\omega_i$, where ω_f is the frequency of the field at the detector; $E_0\equiv|\vec{E}_0|$is the magnitude of the incident field and $\delta\varepsilon_{if}(\vec{k},t)\equiv\widehat{i}\cdot\delta\overleftrightarrow{\varepsilon}(\vec{k},t)\cdot\widehat{f}$, where $\widehat{i}$ identifies a direction perpendicular to the scattering plane and $\widehat{f}$ parallel to it.

The light scattering spectrum, $S(\vec{k},\omega)$, is proportional to the spectral density $I_{if}(\vec{k},\omega)$ of the dielectric tensor time autocorrelation function [44,39]

$$I_{if}(\vec{k},\omega) \equiv \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t}\langle \delta\varepsilon_{if}(\vec{k},t)\delta\varepsilon^{*}_{if}(\vec{k},0)\rangle\,\mathrm{d}t \tag{97}$$

that is

$$S(\vec{k},\omega) = \frac{\vec{k}_f^4|E_0|^2}{32\pi^3\varepsilon_0^2R^2}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t}\langle \delta\varepsilon_{if}(\vec{k},t)\delta\varepsilon^{*}_{if}(\vec{k},0)\rangle\,\mathrm{d}t\,, \tag{98}$$

where the asterisk denotes complex conjugate.

A nematic liquid crystals is an uniaxial medium and its dielectric tensor is of the general form

$$\varepsilon_{\alpha\beta} = \varepsilon_{\perp}\delta_{\alpha\beta} + \varepsilon_a n_\alpha n_\beta\,, \tag{99}$$

where the dielectric anisotropy $\varepsilon_a \equiv \varepsilon_{\parallel} - \varepsilon_{\perp}$ is the difference between the dielectric constants along the director and a direction perpendicular to it. Since for the present model we have chosen $\widehat{n}_0 = (0,0,n_{0z})$ and since we have considered only the transverse variables, it follows that spectrum $S(\vec{k},t)$ is specified by the orientation autocorrelation $\widetilde{\chi}_{22}(\vec{k},\omega)$ only,

$$S(\vec{k},\omega) = \frac{\vec{k}_f^4|E_0|^2}{32\pi^3\varepsilon_0^2R^2}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}\omega t}\langle a_6(\vec{k},t)a_6^{*}(\vec{k},0)\rangle\,\mathrm{d}t\,. \tag{100}$$

From Eq. (88) the normalized equilibrium structure factor is the sum of two Lorentzians,

$$\begin{aligned} S^{eq}(\vec{k},\omega) &\equiv \frac{\langle \widetilde{a}_6^{eq}(\vec{k},\omega)\widetilde{a}_6^{eq}(-\vec{k},-\omega)\rangle}{\langle \widetilde{a}_6^{eq}(\vec{k})\widetilde{a}_6^{eq}(-\vec{k})\rangle} \\ &= \frac{2(2\pi)^4\delta^4(0)}{\gamma_1}k_BT_0\left[\frac{1+\alpha}{\omega^2+\mid\omega_2^{eq}\mid^2} - \frac{\alpha}{\omega^2+\mid\omega_1^{eq}\mid^2}\right]. \end{aligned} \tag{101}$$

Moreover, for values of the frequency ω sufficiently close to the maximum of the Lorentzians, using Eqs. (73) and (74), Eq. (101) reduces to

$$S^{eq}(\vec{k},\omega) \equiv \frac{2(2\pi)^4\delta^4(0)}{\gamma_1}k_BT_0\frac{1+\alpha}{\omega^2+k^4\Gamma(\varphi)}\,, \tag{102}$$

where we have defined $\Gamma \equiv [\frac{1+\alpha(\varphi)}{\gamma_1}]^2(K_2\sin^2\varphi + K_3\cos^2\varphi)^2$. Similarly, from Eq. (93) the normalized nonequilibrium part of $S^{neq}(\vec{k},\omega)$ may be written in the form

$$\begin{aligned} S^{neq}(\vec{k},\omega) &\equiv -2\gamma b_1k_y\frac{2(2\pi)^4\delta^4(0)}{\gamma_1}k_BT_0\frac{\omega(1+\alpha)}{[\omega^2+k^4\Gamma(\varphi)]^2} \\ &= -2\gamma b_1k_yS^{eq}(\vec{k},\omega)\frac{\omega}{\omega^2+k^4\Gamma(\varphi)}\,. \end{aligned} \tag{103}$$

In compact form the normalized dynamic structure factor may then be written as

$$S_0(\omega_0) \equiv \frac{S(\vec{k},\omega)}{S(\vec{k})} \equiv \frac{S^{eq}(\vec{k},\omega)+S^{neq}(\vec{k},\omega)}{S(\vec{k})} = \frac{1}{1+\omega_0^2}\left[1- \mid \nabla P \mid \frac{\omega_0}{1+\omega_0^2}\right], \quad (104)$$

where we have introduced the dimensionless frequency $\omega_0 \equiv \omega/k^2\Gamma^{1/2}(\varphi)$ and the abbreviation

$$\mid \nabla P \mid \equiv \gamma \frac{2b_1 \sin \varphi}{k\Gamma^{1/2}(\varphi)} . \quad (105)$$

From Eq. (104) it follows that the presence of the external pressure gradient introduces an asymmetry in the spectrum and the maximum of the structure factor is displaced with respect to its equilibrium position. To calculate the magnitude of this shift to lowest order in the external pressure gradient, we set $\frac{\mathrm{d}}{\mathrm{d}\omega}S(\omega_0)=0$, which defines the position of the maximum of $S(\omega_0)$ as a function of $\mid \nabla P \mid$. This yields the cubic equation

$$\omega_0^3 - \frac{3}{2} \mid \nabla P \mid \omega_0^2 + \omega_0 + \frac{1}{2} \mid \nabla P \mid = 0 \quad (106)$$

which has two imaginary and one real solutions. The real solution ω_0^r, is plotted as a function of $\mid \nabla P \mid$ in Fig. 2 and it shows that $\omega_0^r = -\frac{1}{2} \mid \nabla P \mid$ in the range $\pm$ 0.1. This

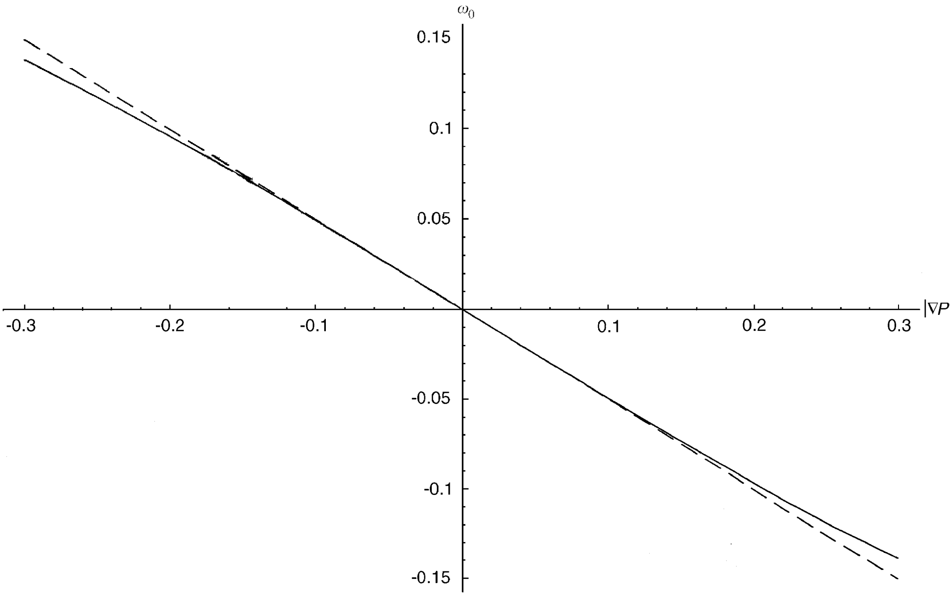

Fig. 2. The real solution ω_0^r plotted as a function of $\mid \nabla P \mid$, (—), in arbitrary units. (- - -) denotes the straight line $\omega_0^r = -\frac{|\nabla P|}{2}$.

result quantifies the approximation made above, i.e., in the range $\omega_0^r = [-0.1, 0.1]$ the analytic expression (104) is valid.

We shall now analyze quantitatively the effect of the Poiseuille flow on $S_0(\omega_0)$ for *MBBA*, which has a nematic phase in the range 24–47 °C and which has the following material parameters: $\lambda = 1.03$, $\rho_0 = 1.029\,\mathrm{g/cm}^3$, $\nu_2 = 41.6 \times 10^{-2}\,\mathrm{g/(cm\,s)}$, $\nu_3 = 23.8 \times 10^{-2}\,\mathrm{g/(cm\,s)}$, $\gamma_1 = 76.3 \times 10^{-2}\,\mathrm{g/(cm\,s)}$, $K_2 = 2.2 \times 10^{-7}$ dyn, $K_3 = 7.45 \times 10^{-7}$ dyn, [35]. To quantify the nonequilibrium effects it is convenient to define

$$\Delta(\vec{k}, \omega) \equiv \frac{S^{neq}(\vec{k}, \omega)}{S^{eq}(\vec{k}, \omega)} = -\gamma 2 b_1 k \sin \varphi \frac{\omega}{\omega^2 + k^4 \Gamma(\varphi)} \; . \tag{107}$$

For a fixed value of $\vec{k}$, the maximum of this quantity occurs for $\omega = \pm\, k^4 \Gamma(\varphi)$ or $\omega_0 = \pm 1$. The maximum of this quantity in terms of dimensionless quantities reads

$$\Delta^*(\omega_0) \equiv \frac{1}{8} \mid \nabla P \mid \; . \tag{108}$$

It represents the largest nonequilibrium shift on the dynamic structure factor. In a similar fashion it is convenient to define

$$\Psi \equiv \frac{S_0(\omega_0 = \omega_0^{\max}) - S_0^{eq}(\omega_0 = 0)}{S_0^{eq}(\omega_0 = 0)} \; , \tag{109}$$

which measures the difference of the size of the peaks with respect to equilibrium. If we take $\gamma = 0.25$, which corresponds to $\mid \nabla p \mid = 2.64 \times 10^{-2}\,\mathrm{atm/cm}$, the nonequilibrium effect on the total dynamic structure factor is shown in Fig. 3. Note that $\Delta^*(\omega_0) \equiv 94.3\%$ and $\Psi \equiv 42.36\%$, which indicate a large effect for rather realistic pressure gradients. This result suggests that this effect on the light scattering spectrum might be observable.

5. Discussion

In summary, we have investigated theoretically the influence of the effects produced by an external pressure gradient on the light scattering spectrum of a thermotropic nematic. To clarify and elaborate on some of these results the following comments may be useful.

It should be emphasized again that the nonequilibrium correction is an odd function of ω that introduces an asymmetry in the shape of the structure factor, shifting the maximum towards the region of negative values of ω. Close to equilibrium the size of the shift depends on the magnitude of the gradient and we may say that these effects could be quite large for relatively low values of the applied real pressure gradient, [32,45]. However, whether these gradients correspond to stable flows or if convection effects may be ignored, remains to be assessed. To our knowledge, there are no experimental data available in the literature to compare with, but their magnitude suggests that they might be experimentally detected. It should be also mentioned that the model has been constructed so that it corresponds

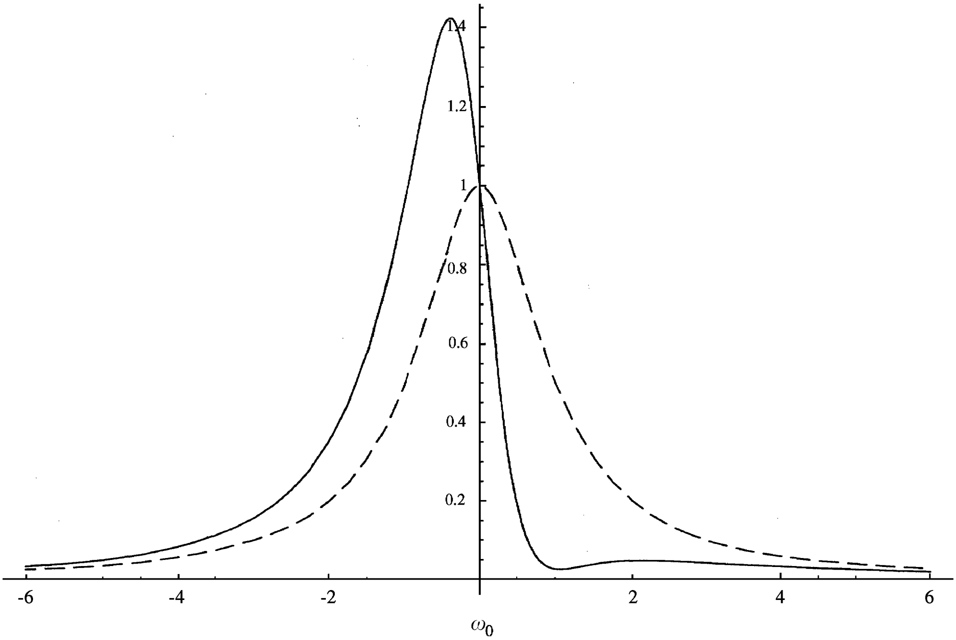

Fig. 3. The structure factor $S_0(\omega_0)$ for *MBBA*, as defined by Eq. (104), vs. ω_0 for $\gamma = 0.25$. (- - -) denotes the equilibrium component and (—) represents the total structure factor. $k_i = 1.14 \times 10^5\,\mathrm{cm}^{-1}$ and $\theta = 1^\circ$. According to Eqs. (108) and (109) $\Delta^*(\omega_0) \equiv 94.3\%$ and $\Psi \equiv 42.36\%$.

to a homodyne experimental arrangement and appropriate to detect the so called mode 1 of the spectrum [39].

Eqs. (102) and (103) imply that $S^{eq}(\vec{k},\omega) \sim k^{-4}$ and $S^{neq}(\vec{k},\omega) \sim k^{-7}$. The differences in these k dependences as compared, for instance, to the dependence of the Rayleigh peak for an isotropic fluid, $S^{eq}(\vec{k},\omega) \sim k^{-2}$, and with the nonequilibrium structure factor of the Brillouin spectrum of a simple fluid under a temperature gradient [24], are related to the nature of the variables involved. For a simple fluid all the variables are conserved, whereas for the liquid crystal δn_x is not a conserved variable. Actually, for the nematic these dependences on k imply a long-range contribution to the spectrum [46].

It should be emphasized once more, that the analysis carried out in this work only included the transverse hydrodynamical variables of a nematic liquid crystal and, as a result, all thermal and coupling effects with the energy were ignored. A more complete analysis should include also the longitudinal variables, $\{\delta\rho(\vec{r},t),\ \delta s(\vec{r},t),\ \delta v_y(\vec{r},t),\ \delta v_z(\vec{r},t),\ \delta n_y(\vec{r},t)\}$, this is essential to determine the complete hydrodynamic modes and the Brillouin spectrum [34].

There are other aspects of the model considered and that are worth investigating. For instance, it is very likely that the calculated correlation functions may develop long-range contributions in the steady state induced by the applied external pressure gradient. It is well known that for many models and systems in nonequilibrium states it has been shown theoretically that the existence of the so called generic scale invariance, is the origin of the long range nature of the correlation functions [21–23]. The investigation of these issues are presently under way [46].

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Appendix

The conservation equations of mass, momentum and the relaxation equation for the director field of a thermotropic nematic liquid crystal read [35,36,41,1],

$$\partial_t \rho + \nabla_i(\rho v_i) = 0 \,, \tag{110}$$

$$\partial_t \sigma + \nabla \cdot (\vec{v}\sigma) + \mathrm{div}\, j_i^s = \frac{R}{T} \,, \tag{111}$$

$$\rho \partial_t v_i + v_j \nabla_j v_i + \nabla_j \sigma_{ij} = 0 \,, \tag{112}$$

$$\partial_t n_i + v_j \nabla_j n_i + Y_i = 0 \,. \tag{113}$$

Here $\rho(\vec{r}, t)$, $s(\vec{r}, t)$, $v_i(\vec{r}, t)$ and $n_\alpha(\vec{r}, t)$ denote, respectively, the local mass density, the entropy density (entropy per unit volume), the hydrodynamic velocity and the director field. The momentum current σ_{ij} and the quasi-current Y_i are defined as

$$\sigma_{ij} = p\delta_{ij} + K_{ijmn} \nabla_n n_m \nabla_j n_l - \frac{1}{2}\lambda_{kji} h_k - v_{ijkl} \nabla_l v_k \tag{114}$$

and

$$Y_i = -\frac{1}{2}\lambda_{kji} \nabla_j v_k + \frac{1}{\gamma_1}\delta^\perp_{ik} h_k \,, \tag{115}$$

where p is the pressure. j_i^s denotes the entropy current and R is the dissipation function, which is positive for irreversible processes and vanishes for reversible ones. It can be interpreted as the energy per unit time and volume dissipated into the microscopic degrees of freedom, R/T represents the entropy production of the nematic. Explicitly [41]

$$\begin{aligned} R = & -\nabla_i (j_i^{\varepsilon D} - T \quad j_i^{sD} - v_j \sigma_{ij}^D) - j_i^{sD} \nabla_i T - \sigma_{ij}^D \nabla_j v_i + h_i \delta^\perp_{ij} Y_j^D \\ = & \frac{1}{2\gamma_1} h_i \delta^\perp_{ij} h_j + \frac{1}{2} v_{ijkl} (\nabla_j v_i)(\nabla_l v_k) + \frac{1}{2}\kappa_{ij} (\nabla_i T)(\nabla_j T) \end{aligned} \tag{116}$$

with $\nabla_i \equiv \partial/\partial x_i$ and where κ_{ij} is the thermal conductivity tensor, $\kappa_{ij} \equiv \kappa_\perp \delta_{ij} + \kappa_a n_i n_j$. Here δ_{ij} is the usual Kronecker's delta and $\delta^\perp_{ik} \equiv \delta_{ik} - n_i n_k$ is a projection operator. $\kappa_a \equiv \kappa_\parallel - \kappa_\perp$ is the anisotropy in the thermal conductivity of the nematic where $\kappa_\perp$

and $\kappa_{\|}$ denote, respectively, its perpendicular and parallel components with respect to the director field. The superscript D in Eq. (116) denotes the irreversible part of the currents. Since $R>0$ for irreversible processes, the coefficients in the above equation should be positive.

In the above equations the fourth order tensor K_{ljmn} depends on the elastic constants K_1 (splay), K_2 (twist) and K_3 (bend) and it is defined in terms of the Levi–Civitta tensor ε_{ijk} by

$$K_{ljmn} = K_1 \delta^{\perp}_{lj} \delta^{\perp}_{mn} + K_2 n_p \varepsilon_{plj} n_q \varepsilon_{qmn} + K_3 n_j n_n \delta^{\perp}_{lm} \,. \tag{117}$$

h_k denotes the so-called molecular field defined in terms of the director by

$$h_k = -K_{kjnl} \nabla_j \nabla_l n_n + \delta^{\perp}_{kq} \left(\frac{\partial}{2\partial n_q} K_{pjkl} - \frac{\partial}{\partial n_q} K_{qjkl} \right) \nabla_l n_k \nabla_j n_p \, ; \tag{118}$$

the viscous tensor v_{ijkl} is

$$\begin{aligned} v_{ijkl} = {} & v_2(\delta_{jl}\delta_{ik+}\delta_{il}\delta_{jk}) + 2(v_1 + v_2 - 2v_3) n_i n_j n_k n_l \\ & + (v_3 - v_2)(n_j n_l \delta_{ik} + n_j n_k \delta_{il} + n_i n_k \delta_{jl} + n_i n_l \delta_{jk}) \\ & + (\gamma_1 - v_2)\delta_{ij}\delta_{kl} + (\gamma_2 - \gamma_1 + v_2)(\delta_{ij} n_k n_l + \delta_{kl} n_i n_j) \,, \end{aligned} \tag{119}$$

where v_i denotes the five nematic viscosity coefficients, v_1, v_2, v_3, γ_1, γ_2, of a nematic in the notation of Harvard [35]. The third-order tensor λ_{kji},

$$\lambda_{kji} \equiv (\lambda - 1)\delta^{\perp}_{kj} n_i + (\lambda + 1)\delta^{\perp}_{ki} n_j \tag{120}$$

depends on the orientational viscosities through $\lambda \equiv -\gamma_1/\gamma_2$.

Considering only linear deviations, $\delta\rho(\vec{r},t) \equiv \rho(\vec{r},t) - \rho_0$, $\delta T(\vec{r},t) \equiv T(\vec{r},t) - T_0$, $\delta v_i(\vec{r},t) \equiv v_i(\vec{r},t)$, $\delta n_i(\vec{r},t) \equiv n_i(\vec{r},t) - n_{0i}$, around the reference state identified by the subscript 0, using the thermodynamic relations

$$\delta p = \left(\frac{\partial p}{\partial \rho} \right)_T \delta\rho + \left(\frac{\partial p}{\partial T} \right)_\rho \delta T \,, \tag{121}$$

$$\delta s = \left(\frac{\partial s}{\partial \rho} \right)_T \delta\rho + \left(\frac{\partial s}{\partial T} \right)_\rho \delta T \tag{122}$$

and taking into account that for small amplitude fluctuations $\delta\vec{n} \cdot \vec{n}_0 = 0$, Eqs. (110)–(113) reduce to

$$\frac{\partial}{\partial t} \delta\rho + \rho_0 \frac{\partial}{\partial x_l} \delta v_l = 0 \,, \tag{123}$$

$$\begin{aligned} & \rho_0 \frac{\partial}{\partial t} \delta v_i + A \frac{\partial}{\partial x_i} \delta\rho + B \frac{\partial}{\partial x_i} \delta T + \frac{1}{2} \lambda^0_{kji} K^0_{ksmr} \frac{\partial^3}{\partial x_j \partial x_s \partial x_r} \delta n_m \\ & \quad - v^0_{ijkl} \frac{\partial^2}{\partial x_j \partial x_l} \delta v_k = 0 \,, \end{aligned} \tag{124}$$

$$\rho_0 C \frac{\partial}{\partial t}\delta T + T_0 B \frac{\partial}{\partial x_l}\delta v_l - \kappa^0_{sj}\frac{\partial^2}{\partial x_s \partial x_j}\delta T = 0\,, \tag{125}$$

$$\frac{\partial}{\partial t}\delta n_i - \frac{1}{2}\lambda^0_{ijk}\frac{\partial}{\partial x_j}\delta v_k - \frac{1}{\gamma_1}\delta^{\perp 0}_{ik} K^0_{kjml}\frac{\partial^2}{\partial x_j \partial x_l}\delta n_m = 0 \tag{126}$$

with $A \equiv (\frac{\partial p}{\partial \rho})_T = c_T^2$, $B \equiv (\frac{\partial p}{\partial T})_\rho = \rho_0 \beta c_T^2$ and $C \equiv \frac{T_0}{\rho_0}(\frac{\partial \sigma}{\partial T})_\rho = c_v$. c_T stands for the isothermal sound speed, c_v is the specific heat at constant volume, $\beta \equiv -(1/\rho)(\partial\rho/\partial T)_p$ is the thermal expansion coefficient and $\kappa_T = (1/\rho)(\partial\rho/\partial p)_T = (\rho c_T^2)^{-1}$ is the isothermal compressibility coefficient. $\delta^{\perp 0}_{ij} \equiv \delta_{ij} - n^0_i n^0_j$ is the linearized projection operator; the linearized tensors λ^0_{kji}, K^0_{ljkm} and v^0_{ijkl}, are given, respectively, by

$$\lambda^0_{kji} \equiv (\lambda - 1)(\delta_{kj} n^0_i - n^0_k n^0_j n^0_i) + (\lambda + 1)(\delta_{ki} n^0_j - n^0_k n^0_i n^0_j)\,, \tag{127}$$

$$\begin{aligned} K^0_{ljkm} \equiv\ & K_1(\delta_{lj}\delta_{km} - n^0_l n^0_j \delta_{km} - \delta_{lj} n^0_k n^0_m + n^0_l n^0_j n^0_k n^0_m) \\ & + K_2 \varepsilon_{plj}\varepsilon_{qkm} n^0_p n^0_q + K_3(n^0_j n^0_m \delta_{lk} - n^0_j n^0_m n^0_l n^0_k)\,, \end{aligned} \tag{128}$$

$$\begin{aligned} v^0_{ijkl} =\ & v_2(\delta_{jl}\delta_{ik} + \delta_{il}\delta_{jk}) + 2(v_1 + v_2 - 2v_3) n^0_i n^0_j n^0_k n^0_l \\ & + (v_3 - v_2)(n^0_j n^0_l \delta_{ik} + n^0_j n^0_k \delta_{il} + n^0_i n^0_k \delta_{jl} + n^0_i n^0_l \delta_{jk}) \\ & + (v_4 - v_2)\delta_{ij}\delta_{kl} + (v_5 - v_4 + v_2)(\delta_{ij} n^0_k n^0_l + \delta_{kl} n^0_i n^0_j)\,. \end{aligned} \tag{129}$$

Following Landau and Lifshitz [1] we now introduce fluctuating components into the momentum current, $\sigma_{ij}(\vec{r}, t)$, the entropy current, $j^s_i(\vec{r}, t)$, and the relaxation quasi-current of the orientation of the nematic, Y_i. These stochastic components are denoted, respectively, by $\nabla_j \Sigma_{ij}(\vec{r}, t)$, $J^s_i(\vec{r}, t)$, $\Upsilon_i(\vec{r}, t)$, and are chosen so that they are zero averaged stochastic processes

$$\langle \Sigma_{ij}(\vec{r}, t)\rangle = \langle \Upsilon_i(\vec{r}, t)\rangle = \langle J^s_i(\vec{r}, t)\rangle = 0$$

satisfying fluctuation–dissipation relations of the form [41,32]

$$\langle \Upsilon_i(\vec{r}, t)\Upsilon_j(\vec{r}\,', t')\rangle = 2k_B T_0 \frac{1}{\gamma_1}\delta^{0\perp}_{ij}\delta(\vec{r} - \vec{r}\,')\delta(t - t')\,, \tag{130}$$

$$\langle J^s_i(\vec{r}, t) J^s_j(\vec{r}\,', t')\rangle = 2k_B \kappa^0_{ij}\delta(\vec{r} - \vec{r}\,')\delta(t - t')\,, \tag{131}$$

where k_B is Boltzmann's constant. Furthermore, if gravity effects are negligible, the linearization process of Eqs. (110)–(113) yields the following complete set of equations:

$$\frac{1}{\rho_0}\frac{\partial}{\partial t}\delta\rho + \nabla_x \delta v_x + \nabla_y \delta v_y + \nabla_z \delta v_z = 0\,, \tag{132}$$

$$\frac{1}{T_0}\frac{\partial}{\partial t}\delta T + \frac{\beta c_T^2}{c_v}(\nabla_x \delta v_x + \nabla_y \delta v_y + \nabla_z \delta v_z) - [\chi_\perp(\nabla_x^2 + \nabla_y^2) + \chi_\parallel \nabla_z^2]\delta T = 0 , \tag{133}$$

$$\begin{aligned}\rho_0 \frac{\partial}{\partial t}\delta v_x &= -c_T^2 \nabla_x \delta\rho - \rho_0 \beta c_T^2 \nabla_x \delta T + [(v_2 + v_4)\nabla_x^2 + v_2 \nabla_y^2 + v_3 \nabla_z^2]\delta v_x + v_4 \nabla_x \nabla_y \delta v_y \\ &\quad + (v_3 + v_5)\nabla_z \nabla_x \delta v_z - \frac{1}{2}(\lambda + 1)(K_1 \nabla_x^2 + K_2 \nabla_y^2 + K_3 \nabla_z^2)\nabla_z \delta n_x \\ &\quad - \frac{1}{2}(\lambda + 1)(K_1 - K_2)\nabla_z \nabla_x \nabla_y \delta n_y ,\end{aligned} \tag{134}$$

$$\begin{aligned}\rho_0 \frac{\partial}{\partial t}\delta v_y &= -c_T^2 \nabla_y \delta\rho - \rho_0 \beta c_T^2 \nabla_y \delta T + v_4 \nabla_y \nabla_x \delta v_x + [v_2 \nabla_x^2 + (v_2 + v_4)\nabla_y^2 + v_3 \nabla_z^2]\delta v_y \\ &\quad + (v_3 + v_5)\nabla_z \nabla_y \delta v_z - \frac{1}{2}(\lambda + 1)(K_1 - K_2)\nabla_z \nabla_x \nabla_y \delta n_x \\ &\quad - \frac{1}{2}(\lambda + 1)(K_2 \nabla_x^2 + K_1 \nabla_y^2 + K_3 \nabla_z^2)\nabla_z \delta n_y ,\end{aligned} \tag{135}$$

$$\begin{aligned}\rho_0 \frac{\partial}{\partial t}\delta v_z = &- c_T^2 \nabla_z \delta\rho - \rho_0 \beta c_T^2 \nabla_z \delta T + (v_3 + v_5)\nabla_z \nabla_x \delta v_x + (v_3 + v_5)\nabla_z \nabla_y \delta v_y \\ &+ [v_3(\nabla_x^2 + \nabla_y^2) + (2v_1 + v_2 - v_4 + 2v_5)\nabla_z^2]\delta v_z \\ &- \frac{1}{2}(\lambda - 1)[K_1(\nabla_x^2 + \nabla_y^2) + K_3 \nabla_z^2]\nabla_x \delta n_x \\ &- \frac{1}{2}(\lambda - 1)[K_1(\nabla_x^2 + \nabla_y^2) + K_3 \nabla_z^2]\nabla_y \delta n_y ,\end{aligned} \tag{136}$$

$$\begin{aligned}\frac{\partial}{\partial t}\delta n_x = &\frac{1}{2}(\lambda - 1)\nabla_x \delta v_z + \frac{1}{2}(\lambda + 1)\nabla_z \delta v_x + \frac{1}{\gamma_1}(K_1 - K_2)\nabla_x \nabla_y \delta n_y \\ &+ \frac{1}{\gamma_1}(K_1 \nabla_x^2 + K_2 \nabla_y^2 + K_3 \nabla_z^2)\delta n_x ,\end{aligned} \tag{137}$$

$$\begin{aligned}\frac{\partial}{\partial t}\delta n_y = &\frac{1}{2}(\lambda + 1)\nabla_z \delta v_y + \frac{1}{2}(\lambda - 1)\nabla_y \delta v_z + \frac{1}{\gamma_1}(K_1 - K_2)\nabla_x \nabla_y \delta n_x \\ &+ \frac{1}{\gamma_1}(K_2 \nabla_x^2 + K_1 \nabla_y^2 + K_3 \nabla_z^2)\delta n_y ,\end{aligned} \tag{138}$$

where γ_2 is an orientational viscosity coefficient; $\chi_\parallel \equiv \kappa_\parallel/\rho c_v$ and $\chi_\perp \equiv \kappa_\perp/\rho c_v$ are, respectively, the parallel and perpendicular components of the thermal diffusivity with respect to the director field; $\nabla_i^2 \equiv \partial^2/\partial x_i^2$ and $\nabla_\perp^2 \equiv \nabla_x^2 + \nabla_y^2$.

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