Equation (59) of PRA **43** pp. 2649-2654 (1991).

As written: (59)

$$\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\eta}(t')-\Gamma_0^{\eta}(t)\right]\varepsilon^{\beta}(t')=\left(1-\frac{1}{2}\delta^{\eta\beta}\right)\left[\Gamma_0^{\eta}(t+\Delta t)-\Gamma_0^{\eta}(t)\right]\left[\Gamma_0^{\beta}(t+\Delta t)-\Gamma_0^{\beta}(t)\right]$$

but it should be

$$\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\eta}(t')-\Gamma_0^{\eta}(t)\right]\varepsilon^{\beta}(t')=\frac{1}{2}\left[\Gamma_0^{\eta}(t+\Delta t)-\Gamma_0^{\eta}(t)\right]\left[\Gamma_0^{\beta}(t+\Delta t)-\Gamma_0^{\beta}(t)\right]$$

The right-hand-side is symmetric in η and β, but the left-hand-side is not. This discrepancy is real only begining with Δt to the 5/2 power. Thus to second order, the second equation is correct as it stands.

Proof:

$$\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\eta}(t')-\Gamma_0^{\eta}(t)\right]\varepsilon^{\beta}(t')=\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\eta}(t')-\Gamma_0^{\eta}(t)\right]\frac{d}{dt'}\left[\Gamma_0^{\beta}(t')-\Gamma_0^{\beta}(t)\right]$$

$$=\left[\Gamma_0^{\eta}(t+\Delta t)-\Gamma_0^{\eta}(t)\right]\left[\Gamma_0^{\beta}(t+\Delta t)-\Gamma_0^{\beta}(t)\right]-\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\beta}(t')-\Gamma_0^{\beta}(t)\right]\varepsilon^{\eta}(t')$$

In addition, aTaylor expansion yields

$$\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\eta}(t')-\Gamma_0^{\eta}(t)\right]\varepsilon^{\beta}(t')\cong 0+\Delta t\left[\Gamma_0^{\eta}(t)-\Gamma_0^{\eta}(t)\right]\varepsilon^{\beta}(t)$$

$$+\frac{1}{2}\Delta t^2\left[\varepsilon^{\eta}(t)\varepsilon^{\beta}(t)+\left[\Gamma_0^{\eta}(t)-\Gamma_0^{\eta}(t)\right]\frac{d}{dt}\varepsilon^{\beta}(t)\right]+O(\Delta t^3)$$

$$=\frac{1}{2}\Delta t^2\varepsilon^{\eta}(t)\varepsilon^{\beta}(t)+O(\Delta t^3)$$

Therefore

$$\int_t^{t+\Delta t} dt'\left[\Gamma_0^{\beta}(t')-\Gamma_0^{\beta}(t)\right]\varepsilon^{\eta}(t')=\frac{1}{2}\Delta t^2\varepsilon^{\eta}(t)\varepsilon^{\beta}(t)+O(\Delta t^3)$$

too!!

Putting this into the first line of the proof together with the second line justifies the corrected formula. QED